The Nonparametric Metadata Dependent Relational Model

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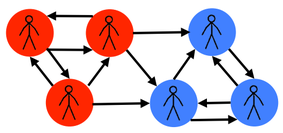
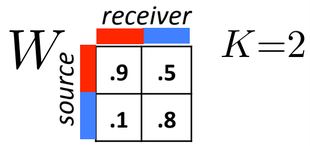
Our Contributions

- **Nonparametric** mixed membership
  - Unbounded number of communities
  - Retrospective MCMC: ***No Truncation***
- **Metadata** informs latent structure
  - Upstream inclusion of metadata leads to recovery of ***interpretable*** communities

Stochastic Block Models

Unsupervised community discovery from observed network edges [Wang JASA 1987]

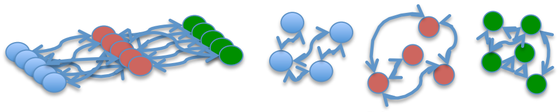




$K=2$

- Assign each node to **one** latent block/community
- Predict edge presence from block assignments of *source* and *receiver* nodes

Various Network Structures Possible



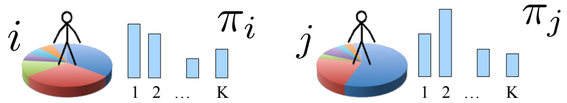
Multipartite Disconnected

Infinite Relational (IRM)

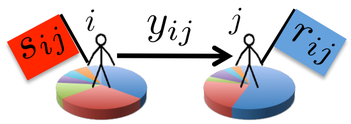
- **Unbounded** number of blocks K, [Kemp AAAI 2006] via Chinese Restaurant Process
- Each node assigned to **one** block

Mixed Membership (MMSB)

- **Finite** number of blocks K, must be specified *a priori* [Airoldi JMLR 2008]
- Each node has **distribution** over blocks



- Each *directed* node pair



Source Block Assignment $s_{ij} \sim \text{Cat}(\pi_i)$

Receiver Block Assignment $r_{ij} \sim \text{Cat}(\pi_j)$

Binary Edge Indicator $y_{ij} \sim \text{Bern}(W_{s_{ij} r_{ij}})$

Using Metadata

Downstream

[Miller NIPS 2009]

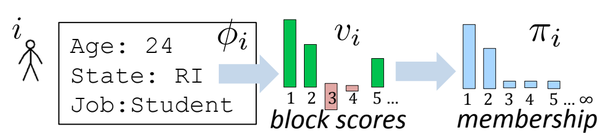
Metadata regression in edge likelihood

$y_{ij} \sim \text{Bern}(W_{s_{ij},r_{ij}} + \eta^T \phi_i)$

Recovered communities **less interpretable,** just explain residual noise

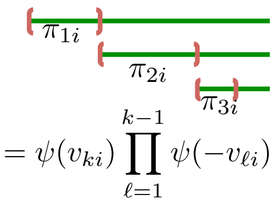
Upstream

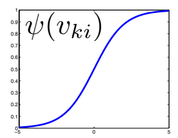
Metadata informs node membership, creates **meaningful communities**



Logistic stick breaking allows ϕ_i to inform π_i

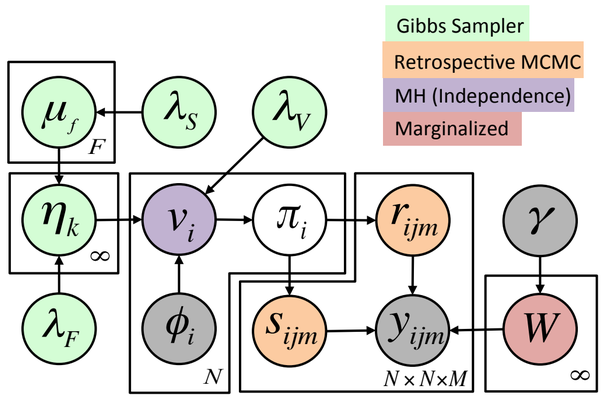
$v_{ki} \sim N(\eta_{:k}^T \phi_{:i}, \lambda_V^{-1})$





$\pi_{ki} = \psi(v_{ki}) \prod_{\ell=1}^{k-1} \psi(-v_{\ell i})$

NMDR Graphical Model



Link presence probability $W_{k\ell} \sim \text{Beta}(\gamma_a, \gamma_b)$

Mean metadata weight $\mu_f \sim N(0, \lambda_S^{-1})$

Metadata regression weight $\eta_{fk} \sim N(\mu_f, \lambda_F^{-1})$

Precision parameters have Gamma priors $\lambda_S, \lambda_F, \lambda_V$

Retrospective MCMC

- Dynamically add/delete blocks across iterations
- Only instantiate K blocks **actively used**
- Create parameters for new blocks as needed

EXAMPLE: Sampling $s_{ij} | \pi_i, y_{ij}, r_{ij} = \ell$

*Typical Gibbs update, with extra term for **new** block*

Counts of present/absent edges with communities *k,l* excluding edge *i,j*

$$\rho_k \propto \pi_{ki} \frac{(A_{k\ell}^{-ij} + \gamma_a)^{y_{ij}} (B_{k\ell}^{-ij} + \gamma_b)^{1-y_{ij}}}{A_{k\ell}^{-ij} + B_{k\ell}^{-ij} + \gamma_a + \gamma_b}$$

$$\rho_{K+1} \propto \left[1 - \sum_{k=1}^{K} \pi_{ki}\right] \frac{\gamma_a^{y_{ij}} \gamma_b^{1-y_{ij}}}{\gamma_a + \gamma_b}$$

remaining "stick" mass

marginal likelihood for empty block

$s_{ij} \sim \text{Mult}(\rho)$

$s_{ij} \leq K$? — *NO*: add ***new*** block

Sample new params from prior

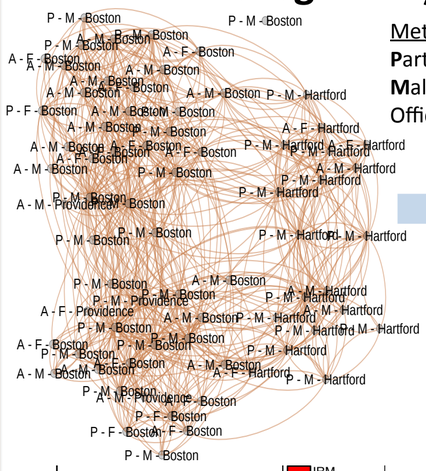
$\eta_{:s_{ij}} \quad v_{s_{ij}i}$

Draw termination indicator

$\omega \sim \text{Bern}(\psi(v_{s_{ij}i}))$

ω=1? — *YES*: $s_{ij} \leftarrow s_{ij} + 1$ — *NO*: ***DONE***

Lazega Lawyers Analysis

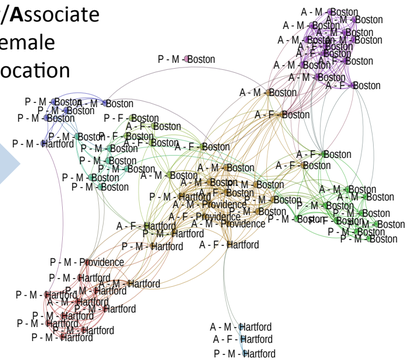


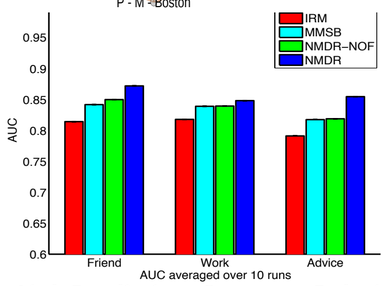
Metadata

Partner/**A**ssociate

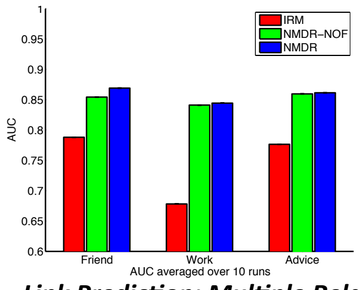
Male/**F**emale

Office Location





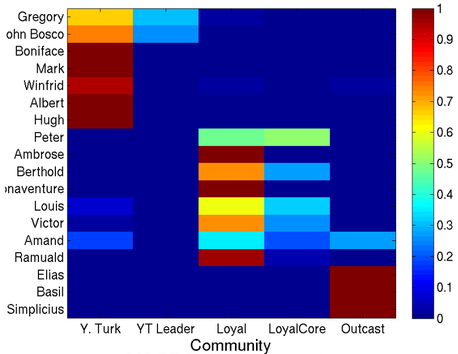
Link Prediction: Separate Relations



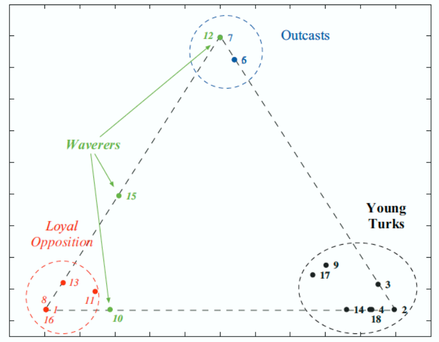
Link Prediction: Multiple Relations

Sampson Monastery Analysis

[Airoldi JMLR 2008]

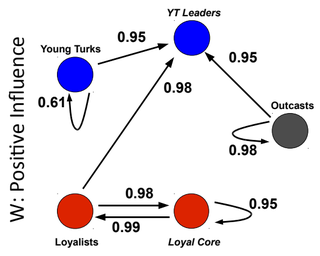


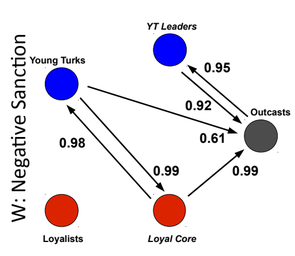
NMDR Communities



1 Ambrose
2 Boniface
3 Mark
4 Winfrid
5 Elias
6 Basil
7 Simplicius
8 Berthold
9 John Bosco
10 Victor
11 Bonaventure
12 Amand
13 Louis
14 Albert
15 Ramuald
16 Peter
17 Gregory
18 Hugh

MMSB Communities (fixed K=3)





NMDR Block-to-Block Link Probabilities